NYSFEA ESSAY CONTEST

Since the inception of the N.Y.S.F.E.A.'s annual essay contest in 1998, five such contests have been held and the announcements for the sixth have been sent out. The Association is grateful to all past sponsors and to those individuals who generously gave their time and energy to guarantee the success of each of these events; and most of all the participants who deserve a special thanks for without them there would be no contest. The following is a list of the institutions which have provided financial support for the awards presented at the Association's annual meetings:

1998 SUNY Oneonta
1999 SUNY Binghamton
2000 SUNY Geneseo
2001 SUNY Cortland
2002 Hofstra University
2003 Siena College (forthcoming)

In addition, the Association is pleased to announce the winners for the third and fourth essay contest. The third essay contest was sponsored by SUNY Geneseo and the annual meeting of the Association was held at SUNY New Paltz (April 7-8, 2000). The winners of the essay contest gave their presentations on April 8, 2000. The first Prize was awarded to Jana Gardner from Manor School, Honeoye Falls-Lima for the essay "Piaget, Constructivism, and Mathematics Education." The Second Prize was awarded to Jane Miller from York Central School for the essay "Literacy and Communication: Understanding and Applying the Theories of Vygotsky." Finally, the Third Prize was awarded to Kristi Fragnoli, a graduate student at SUNY-Binghamton for the essay, "Why Do They Do The Things They Do? A Study of Theory, Practice and Change within the Classroom."

The fourth essay contest was sponsored by SUNY Cortland where the annual meeting of the Association was held on April 6-7, 2001. The winners Lindsey Barton, Lee Marley and Kathrine Valdran were all students at SUNY Geneseo and addressed the topic "Theory and Practice - Expanding Environment in both the College and the Elementary Classroom," at the meeting on April 7, 2001. It was the view of the readers that although the essays were clearly worthy of merit none were deserving of a first prize and, consequently, no essay from the 4th competition will appear in *Educational Change*. However, financial awards were distributed according to the readers recommendations. Last, but not least, a special thanks to the co-ordinators and readers of the essays. The 3rd contest was coordinated by Jane Fowler Morse and readers were Sue Books, Eduardo Duarte and Anthony Roda while the fourth was co-ordinated by Sue Books with Eduardo Duarte, Anthony Roda and Kim Scott as readers.

The following is Jana Gardner's First Prize Essay from the third contest.

PIAGET, CONSTRUCTIVISM, AND MATHEMATICS EDUCATION

Jana Gardner

Contemporary teachers suffer from a perpetual problem that undermines their day to day operations in preparing youngsters for a productive role in society. In practical terms, educators are presented with the realities of covering a curriculum with outdated material, burgeoning classes with a multitude of behavior issues, unsupportive parents, and an administration that seems to spend more time putting out fires than providing leadership. In other words, teachers have to deliver the goods in an effective manner in a sometimes impossible and often less than optimal environment. In contrast to this scenario, educators are also mindful of what they would *like* to see happening: a scholastic nirvana in which they can implement effectively the educational theories and philosophies that seem phenomenal on paper and in the workshop setting. It is a frustrating endeavor to try to bring this desired objective to fruition. Constructivism is a viable solution to this dilemma, especially for the mathematics educator. This approach offers intrinsically motivating activities soundly based in developmental psychology, empowering the learner with a self-created body of knowledge. In this paper, I will touch on the merits of Piaget's view, outline the basics of his theory, describe how constructivism evolved from this, and conclude with some considerations on implementing constructivism in the classroom.

Giambattista Vico, an eighteenth century Italian philosopher, says that, "Humans can only understand what they themselves have constructed." (Microsoft Bookshelf CDROM, 1995). This is a central point, for, as educators, we must decide the wherefores and the why of how our students will arrive at knowledge. Do we feel that we, as teachers, are the gatekeepers of learning, and that knowledge is some separate, absolute truth? Or, like Kant and Dewey, does an experience have to be perceived and "done actively" in order to hold any relevance for the individual? Our philosophical beliefs drive the programs that we implement in our classrooms and are the keel that keeps us pointed toward our destination in the roughest of seas. With the pressures brought to bear on today's classroom teachers, it is incumbent upon us to "set sail" in a seaworthy craft, namely, Piaget's theory of stages allied with a constructivist view of learning.

Just as epistemology has advanced over time, from Plato's search for "the forms," through Bacon's tossing out of the idols in favor of objectivity, and onto current views that take into account the knower's perspective, likewise the science of psychology has progressed to a new understanding of the human mind, and especially that of the child. In the classroom, pedagogical, authoritarian styles of learning were being countered with modern methods. On a parallel path and time frame with these two advances, the perception of mathematics changed from being incidentally acquired while in pursuit of a separate discipline, i.e., in the study of science, to a worthwhile pursuit of its own, largely due to increased industrialization and technological advances. (Gadanidis, 1994).

Drill and practice were thought to be the best ways for students to learn mathematics. (Unfortunately, this doctrine is still well entrenched.) The premise that "meaning gets in the way of efficient computation and that students cannot deduce mathematical rules from examples and other rules..." and that "bonds between stimuli and responses are strengthened through exercise," was maintained by E. L. Thorndike, considered the founder of psychology of mathematics education. (Gadanidis, 1994). Progressive thinkers and educators, among them John Dewey, Jerome Bruner, and Jean Piaget challenged this teacher-centered, adult-oriented behavioristic view. Piaget was the first major contemporary to develop a clear idea of what would come to be called constructivism, which applied knowledge of children's cognitive development to pedagogy in the classroom. For over 60 years, Jean Piaget conducted a program of research that has greatly affected our understanding of child development. Piaget called his theory "genetic epistemology" because he was interested in how knowledge developed in human organisms. With a background in both biology and philosophy, concepts from both disciplines influenced his work. His research had one unique goal: how does knowledge grow? "His answer is that the growth of knowledge is a progressive construction of logically embedded structures superseding one another by a process of inclusion of lower less powerful means into higher and more powerful ones up to adulthood. Therefore, children's logic and modes of thinking are initially entirely different from those of adults." (Boatman, 1998). More simply put, Piaget's constructivism is based on his view of the psychological development of "cognitive structures" of children. Cognitive structures are patterns of physical or mental action that underlie specific acts of intelligence and correspond to stages of child development. (McKeachie, 1994). Piaget's stages of development contain within them elements which are conducive to change. Each stage defines one set moment in child development and then progresses to another level or stage. The resulting well-structured model presents a situation to be overcome or an action to be performed so an individual can move to another progressive, higher stage. (Boatman, 1998).

Essentially, Piaget's theory shows intellectual development as occurring in four distinct periods or stages. Development is continuous, but the intellectual operations in the different periods are distinctly different. Children progress through the four stages in the same order, but at different rates. The stages do not abruptly end but trail off and merge with the next.

Sensorimotor (age 0-2). Intelligence takes the form of motor actions. In this period, a child learns about his or her relationship to various objects, including learning a variety of fundamental movements and perceptual activities. Knowledge involves the ability to manipulate objects such as holding a bottle. In the latter part of this period, the child starts to think about events that are not immediately present.

Preoperational (age 2-7). Piaget had divided this state into the preoperational and the intuitive phase. In this preoperational stage, children use language and try to make sense of the world. They need to test thoughts with reality and do not appear to be able to learn from generalizations. Knowledge becomes intuitive later in this phase; the child moves away from drawing conclusions based solely on experiences with concrete objects. However, these conclusions are based on rather vague impressions and perceptual judgements. It becomes possible to carry on a conversation with a child. They develop the ability to classify objects on the basis of different criteria, and also learn to count and use the concept of numbers.

Concrete operational (age 7-12). In this stage, a child can do mental operations, but only with real (concrete) objects events or situations. Logical reasons are understood. For example, a concrete operational person can understand the need to go to bed early when it is necessary to rise early the next morning. Piaget thought that the concrete operational stage ended at age eleven or twelve. There is now considerable evidence that these ages are the end limits to this stage and that many adults remain in the stage throughout their lives.

Formal operational (age 12+). A formal operational thinker can do abstract thinking and starts to enjoy abstract thought. He or she can formulate hypotheses without actually manipulating concrete objects, and when more adept can test the hypotheses mentally. The formal operational thinker can generalize from one kind of real object to another and onto an abstract notion. Other abilities include the capacity to think ahead and plan, along with the capability of metacognition. (McKeachie, 1994).

Constructivist learning theory asserts that students acquire new knowledge through the process of *adaption*. Adaption is a change in cognitive structures or schemas, which are simply ways in which individuals have generalized or worked out things. Adaption has two components. Assimilation involves the interpretation or incorporation of events in terms of existing cognitive structures, whereas accommodation refers to the changing or modification of existing schemes to make sense of the environment. (Boatman, 1998). Cognitive development consists of a constant effort to try to adapt to the environment through these two processes. The experience of a contradiction, similar to Dewey's "felt difficulty," is known as cognitive disequilibrium. It is the overcoming of such a contradiction that results in new constructions. New, as well as existing, knowledge is transformed as students construct more inclusive schemas of understanding. (Gadanidis, 1994). A classic example, rendered even more significant because of the notable ability of the players involved, was discussed by Piaget in a 1968 lecture to illustrate this process. He explained how Einstein's theory of relativity caused a certain amount of "cognitive disequilibrium" in the physics community.. It required a rethinking – assimilation and accommodation – of long accepted notions among some very powerful thinkers.

While the cognitive development stages identified by Piaget are associated with characteristic age spans, they can vary for every individual. In addition to the four stages, Piaget also describes three types of knowledge that form somewhat of a hierarchy, with the base being physical knowledge, followed by logical-mathematical thinking, and finally social-arbitrary ability. The tendency is for the progression to proceed from simple and self-oriented to more sophisticated and objective.

At this point, I think it is important to make a distinction and establish the correlation between genetic epistemology and constructivism. The former is a theory... a rationalization for why the two year old solves a problem one way, the seven year old another, and the thirteen year old in a completely different fashion. For a variety of reason, constructivism is hard to categorize as a method, a theory, or an epistemology. So, what is the link between constructivism and genetic epistemology? What is it about Piaget that constructivists love? Although few writers have expressed a definitive connection between the theory and the practice, I believe that this convergence lies in the concrete operations stage. Wheatley's statement that "knowledge originates in the learner's activity performed on an object" substantiates my reasoning. Concrete operations is the realm where the learner can *prove* a concept to himself. He continues, "Knowledge is not passive, cannot be given or sent, but must be actively built up by the individual." (Wheatley, 1991).

Constructivism is contrasted to any approach that has students acquiring knowledge in a passive process, i.e., knowledge is passed unchanged from teacher to student. Constructivists would say that even in a lecture situation, the student would still ultimately construct his or her own understanding. (Classroom Compass, 1995).

The significance this philosophy holds for educational practice is that the educator must realize that Piaget's theory is geared toward children, not adults, and centered on the actions of the child, not the teacher. Therefore, it is imperative that teachers must, first, understand the steps of development of a child's mind and, second, be willing to relinquish center stage and take on the role of facilitator or coach. This second task can actually be more difficult to carry out than one would think, for it requires a certain security on the part of the teacher, who must possess enough confidence in his or her function in the learning process that he or she is willing to step back.

What is the teacher's role? In short, to provide "fertile soil" for the students. That is, to bring about the classroom conditions where initiative and independence are encouraged and students gain their own intellectual identity. Open-ended questions are asked requiring more than a simple factual response, while encouraging connection and summarization (with ample time for reflection). Communication is encouraged in a variety of ways – dialogue, charts, maps – to exchange, refine, and reinforce ideas; activities are done with real world stuff – raw data, primary sources, manipulatives, and interactive materials. One would expect students to demonstrate skills like generating ideas, posing problems, hypothesizing, testing ideas, generating multiple solution strategies, and evaluating the consequences of mathematical calculations. (Classroom compass). The students are searching for viable solutions to the problem they are facing... not necessarily an ultimate answer, but one that works for them within the given constraints and at their particular cognitive stage.

The magnitude of information that students potentially have to deal with – in general, and specifically within the field of mathematics – is enormous. It is ludicrous for the educator to assume a "dispense of wisdom" stance, viewing the task of mathematics education as a "codified body of knowledge to be taught." (Wheatley, 1991). The task of learning mathematics is too immense to use a rote method, where the teacher explains the process using the left-hand pages in the text and the students practice problems on the right hand side. In this case, 9 + 5 and 5 + 9 would have to be learned as individual skills. For the constructivist, understanding the underlying patterns and inferring from one relationship to another is what mathematics is about. The emphasis should be on understanding integrated principles and unifying structures. Otherwise, students are just reciting mathematical facts.

Additionally, the question is not only whether the students construct understanding of mathematical concepts but also how good their constructions are. Thus, a “constructivist teacher’s emphasis is on creating learning environments that help students create good schemas of mathematical understanding.” (Gadanidis, 1994). It is a challenge for the instructor to be fully cognizant of the essential concepts and skills and to then be able to present them in a problematic way. A thorough knowledge of the discipline as well as the students’ cognitive developmental stage are critical. “Problem centered learning requires considerable restructuring of course materials as well as different metaphors and images for teaching and learning. Conventional textbooks designed to be used in an explain-and-practice mode are a poor source for tasks.” (Wheatley, 1991). Needless to say, it will take a dedicated individual to overcome these obstacles.

To illustrate a constructivist approach to mathematics, I chose one of the first activities with which I begin the school year of my fifth grade math students. According to Piaget, fifth graders would be well into the concrete stage and approaching the beginning of the abstract thinking stage. A well designed lesson is going to incorporate this, ensuring that, in addition to a hands-on aspect, they will have to stretch their understanding to apply the concept symbolically or abstractly. This unit has students exploring the factors and multiples of the “landmark” numbers of 100, 1,000, and 10,000. They will build the numbers in a variety of ways, and use their knowledge of relationships among these numbers to develop computational strategies. Emphasis is on reasoning about number characteristics using terms such as multiple, factor, even, odd, prime, and square. Student pairs begin by picking a number between 10 and 30. They are instructed to count out that number of small tiles and create as many “rectangles” as possible (for example, the choice of 18 yields 1 by 18, 2 by 9, and 3 by 6), using graph paper to record their examples. Through questioning and sharing, the students develop the idea of factors and its reciprocal concept of *multiples*. It is discovered that *even* numbers can be constructed in rectangles two tiles wide and that only *square* numbers can be formed into a rectangle of equal dimensions. Number puzzles are then used to reinforce and practice. Students work on puzzles in small cooperative groups, with the support of working definitions, charts, tiles, and other ideas we have developed to this point.

A typical puzzle might look like this:

- My number is a square number.
- My number is even.
- My number is less than 100.
- My number is prime.

Students develop and compare different strategies for solving these puzzles. The instructor will periodically draw the group together to share solution strategies and record findings on chart paper, asking questions such as, “How did you go about finding the answer? How did you decide that your answer was correct? Why did you think this puzzle was impossible?” Students’ responses might be drawn, explained verbally to a partner, written in a journal, or constructed from manipulatives. Homework or an assessment might consist of giving the student the first three clues of a puzzle and asking them to write a clue that gives the puzzle just one answer. The unit progresses from here, with factor pairs of 1000 beginning the next level of exploration, highlighted by representative ways of multiplication. At one point, the multiplication algorithm is *not* allowed, forcing the student to construct – either on their own or with some support – and utilize alternative ways of figuring out just what 26 x 47 means.

Educators have long accepted that most students at the elementary school level function at the concrete operational stage and so use concrete representations of the concepts being studied, whereas they have assumed that high school students work at the formal operational stage and, therefore, are able to utilize symbolic and abstract representations. However, research shows that 50 percent of students sixteen and older function at the concrete operational level. (Gadanidis, 1994). This would indicate that the use of concrete representations would be advantageous in enabling the middle school and high school student to internalize and truly understand the mathematics principles at hand. Dr. Steve West, a mathematics professor at SUNY Geneseo, routinely uses manipulatives for his college freshman as they struggle to get a grip on the topic on which they are working. As a matter of fact, students who are taking the mathematics courses required for an education degree are required to purchase a kit of manipulatives including geo-blocks, Cuisenaire rods, and geometrical pattern blocks. These students come away from their preparation with a truer grasp of the concept. (Personal communication, April 29, 1999). I only wish that my calculus professors had given me models and objects to manipulate and discuss with my peers instead of some arcane formula for finding the balance point of a solid object.

In this same line of thought, the educator must realize that not only is the progression to formal operations not a guarantee, but that progression from one stage to the next might resemble more of a waltz, with one step forward and two steps back. Furthermore, formal operations might be achieved in one subject area and not in another. At this juncture, a process known as scaffolding would be the appropriate measure for the teacher to take: "Scaffolding allows students to perform tasks that would normally be slightly beyond their ability without that assistance and guidance from the teacher. Appropriate teacher support can allow students to function at the cutting edge of their individual development." (Classroom Compass, 1995). For example, a simple 3-column chart with the headings *sometimes, never* and *always* can help a student sort polygons by their attributes, providing a foundation and framework from which to build their knowledge.

Jana Gardner

Contrary to common assumptions, there is a social learning element to constructivist mathematics education. To use Wheatley's term, the learner must test the "viability" of his solution. After the first few steps, mathematics can no longer be learned by means of interaction with a 'concrete' environment, but requires the 'confrontation' of the student's cognitive model with that of another student or teacher... again, ensuring that the child has not constructed false knowledge, but has built a good schema. (Gadanidis, 1994). "The very act of formulating an expression of their views promotes reflection which then leads to revision. It is not unusual for people to modify their position once it has been communicated to others in a small group setting." (Wheatley, 1991).

In closing, I would ask that those of us who are engaged in shaping the future through the students in our classrooms reflect upon or revisit the foundations of how we teach. I came to the realization that I had been philosophically operating on a hodgepodge, using practices that intuitively felt like the right thing to do (perhaps because much of the time I act like a typical kid – kinesthetics and curious), but leaving me unable in a professional sense to articulate upon exactly what this intuition was based. As a result of this review of Piaget's epistemology and the underlying characteristics of constructivism, I now understand why this active approach appealed to me so, and why it merits strong consideration by all educators. It can be nicely summarized by looking at the synonyms of the word scheme from which schema is derived: categorizing, planning, engineering, building, and... *doing*.

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