G 1.5 (1)

we could execute the previous statement and integrate by parts to solve or return to the original ~~sta~~ expression of the function. when $f(x) = -f(-x)$ the function is odd. The integral of odd functions over symmetric limits is zero

$$\langle x \rangle = \int_{-\infty}^{\infty} \Psi^* x \Psi \, dx = A^2 \int_{-\infty}^{\infty} x \, e^{-2\lambda |x|} \, dx$$

$$\left.\begin{aligned} f(x) &= x \, e^{-2\lambda |x|} \\ f(-x) &= -x \, e^{-2\lambda |x|} \end{aligned}\right\} \text{odd function}$$

$$\langle x \rangle = 0$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \Psi^* x^2 \Psi \, dx = A^2 \int_{-\infty}^{0} x^2 e^{2\lambda x} dx + A^2 \int_{0}^{\infty} x^2 e^{-2\lambda x} dx$$

$$\int_{-\infty}^{0} \underset{f}{x^2} \underbrace{e^{2\lambda x} dx}_{dg} \qquad dg = 2x\,dx \qquad g = \frac{1}{2\lambda} e^{2\lambda x}$$

$$\int f \, dg = -\int g \, df + gf$$

$$\int_{-\infty}^{0} x^2 e^{2\lambda x} dx = -\int_{-\infty}^{0} \frac{2x}{2\lambda} e^{2\lambda x} dx + \underbrace{\left. \frac{x^2}{2\lambda} e^{2\lambda x} \right|_{-\infty}^{0}}_{0-0=0}$$

$$= -\frac{1}{\lambda} \int_{-\infty}^{0} x e^{2\lambda x} dx$$

6 1.5 (a)

now $\int_{-\infty}^{0} x\, e^{2\lambda x}\, dx$ $\qquad f = x \quad dg = e^{2\lambda x} dx$

$\qquad df = dx \quad g = \frac{1}{2\lambda} e^{2\lambda x}$

$$\int_a^b f\, dg = -\int_a^b g\, df + gf\Big|_a^b$$

$$\int_{-\infty}^{0} x e^{2\lambda x} dx = -\frac{1}{2\lambda}\int_{-\infty}^{0} e^{2\lambda x} dx + x e^{2\lambda x}\Big|_{-\infty}^{0}$$

$[0-0]$

$$= -\frac{1}{(2\lambda)^2} e^{2\lambda x} = \frac{-1}{(2\lambda)^2}[1-0]$$

$$\therefore \int_{-\infty}^{0} x^2 e^{2\lambda x} dx = -\frac{1}{\lambda}\left[-\frac{1}{(2\lambda)^2}\right] = \frac{1}{4\lambda^3}$$

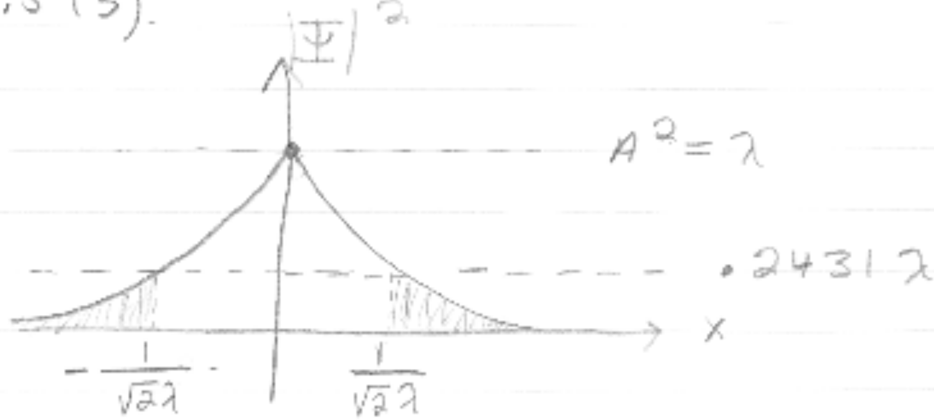
$$\langle x^2 \rangle = A^2\left(\frac{1}{4\lambda^3}\right) + \underline{A^2\left(\frac{1}{4\lambda^3}\right)}$$

I did not explicity do $A^2 \int_0^{\infty} x^2 e^{-2\lambda x} dx$ but method is the same as for the first term.

$$\langle x^2 \rangle = (\lambda)\left(\frac{1}{2\lambda^3}\right) = \frac{1}{2}\frac{1}{\lambda^2}$$

$$\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle} = \sqrt{\frac{1}{2}\frac{1}{\lambda^2}} = \frac{1}{\sqrt{2}\,\lambda}$$

G 1.5 (3).



$$|\Psi|^2 = \Psi^*\Psi = A^2 e^{-2\lambda x} \qquad x > 0$$

$$= A^2 e^{-2\lambda\left(\frac{1}{\sqrt{2}\lambda}\right)}$$

$$= \lambda e^{-2/\sqrt{2}} = \lambda e^{-\sqrt{2}} = .2431\lambda$$

Probability outside range $\langle x \rangle \pm \sigma$ is the area of the shaded regions

$$P_{>\sigma} = \int_\sigma^\infty \Psi^*\Psi \, dx$$

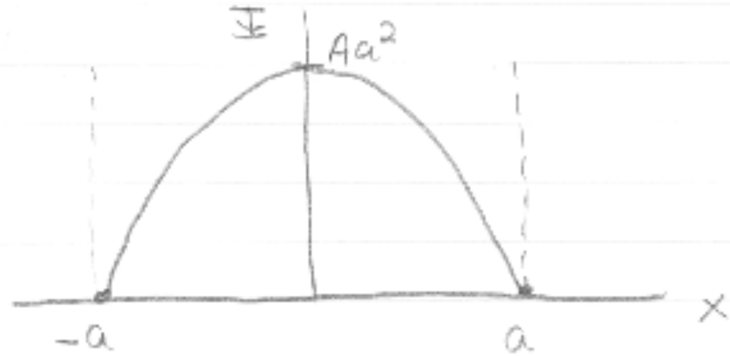
$$P_{>\sigma} = 2\int_\sigma^\infty A^2 e^{-2\lambda x} dx$$

$$P_{>\sigma} = 2\lambda \left.\frac{e^{-2\lambda x}}{-2\lambda}\right|_\sigma^\infty = 0 - -e^{-2\lambda\sigma}$$

$$= e^{-2/\sqrt{2}} = .2431$$

G 1.17 (1)

$$\Psi(x,0) = \begin{cases} A(a^2-x^2) & -a \le x \le a \\ 0 & \text{otherwise} \end{cases}$$



a) $$\int_{-\infty}^{\infty} \Psi^* \Psi = 1$$

$$\int_{-a}^{a} A(a^2-x^2) A(a^2-x^2)\,dx = 1$$

$$A^2 \int_{-a}^{a} (a^4 - 2a^2x^2 + x^4)\,dx = 1$$

$$A^2 \left(2a^5 - 4\frac{a^5}{3} + 2\frac{a^5}{5} \right) = 1$$

$$A^2 a^5 \left(\frac{30}{15} - \frac{20}{15} + \frac{6}{15} \right) = 1$$

$$A = \sqrt{\frac{15}{16}}\, a^{-5/2}$$

G 1.17 (a)

b)

$$\langle x\rangle = \int_{-\infty}^{\infty} \Psi^* x \Psi \, dx$$

$$\langle x\rangle = \int_{-a}^{a} \underbrace{x A^2(a^2-x^2)^2}_{\text{odd integrand}} dx$$

odd integrand
$f(-x) = -f(x)$
about symmetric limits

$$\langle x\rangle = 0$$

c)

$$\langle p\rangle = \int_{-\infty}^{\infty} \Psi^* \frac{\hbar}{i}\frac{\partial}{\partial x} \Psi \, dx$$

$$= \int_{-a}^{a} A(a^2-x^2) \frac{\hbar}{i}\frac{\partial}{\partial x} A(a^2-x^2) \, dx$$

$$= \frac{\hbar A^2}{i} \int_{-a}^{a} \underbrace{(a^2-x^2)(-2x)\, dx}_{\text{odd integrand symmetric limits}}$$

odd integrand symmetric limits

$$\langle p\rangle = 0$$

61.17 (3

d) $\langle x^2 \rangle = \int_{-\infty}^{\infty} \Psi^* x^2 \Psi \, dx$

$$= A^2 \int_{-\infty}^{\infty} x^2 (a^2 - x^2)^2 \, dx$$

$$= A^2 \int_{-a}^{a} (a^4 x^2 - 2a^2 x^4 + x^6) \, dx$$

$$= 2A^2 \left(\frac{a^7}{3} - \frac{2}{5} a^7 + \frac{a^7}{7} \right)$$

$$= 2A^2 a^7 \left(\frac{1}{3} - \frac{2}{5} + \frac{1}{7} \right)$$

$$= 2 \frac{15}{16a^5} a^7 \left(\frac{35}{105} - \frac{42}{105} + \frac{15}{105} \right)$$

$$= 2 \frac{15}{16} a^2 \frac{8}{105} = \frac{a^2}{7}$$

$\langle x^2 \rangle = a^2/7$

e) $\langle p^2 \rangle = -\hbar^2 \int_{-\infty}^{\infty} \Psi^* \frac{\partial^2}{\partial x^2} \Psi \, dx$

$$\langle p^2 \rangle = -\hbar^2 \int_{-a}^{a} A(a^2 - x^2) \frac{\partial^2}{\partial x^2} A(a^2 - x^2) \, dx$$

$$\langle p^2 \rangle = -\hbar^2 A^2 \int_{-a}^{a} (a^2 - x^2)(-2) \, dx$$

G 1.17 (4)

e) $\langle p^2 \rangle = 2\hbar^2 A^2 \int_{-a}^{a} (a^2 - x^2)\,dx$

$$\langle p^2 \rangle = 4\hbar^2 A^2 \left(a^3 - \frac{a^3}{3}\right)$$

$$\langle p^2 \rangle = 4\hbar^2 \frac{15}{16a^5} \frac{2a^3}{3} = \frac{\hbar^2}{a^2} \frac{10}{4} = \frac{5}{2} \frac{\hbar^2}{a^2}$$