G 1.3 pg 1

$$\rho(x) = A e^{-\lambda(x-a)^2}$$

$$\int_{-\infty}^{\infty} \rho(x)\, dx = 1$$

$$\int_{-\infty}^{\infty} A e^{-\lambda(x-a)^2} dx = 1 \qquad u = x-a \qquad du = dx$$

$$\underbrace{A \int_{-\infty}^{\infty} e^{-\lambda u^2} du}_{} = 1 \qquad \int_{-\infty}^{\infty} e^{-a^2 u^2} du = \frac{\pi^{1/2}}{a} \qquad a = \sqrt{\lambda}$$

$$A \quad \frac{\pi^{1/2}}{\lambda^{1/2}} = 1$$

$$\therefore \quad A = \frac{\lambda^{1/2}}{\pi^{1/2}}$$

$$\langle x \rangle = \int_{-\infty}^{\infty} x\, \rho(x)\, dx = A \int_{-\infty}^{\infty} x e^{-\lambda(x-a)^2} dx$$

$$\langle x \rangle = A \int_{-\infty}^{\infty} (u+a) e^{-\lambda u^2} du$$

$$\langle x \rangle = \underbrace{A \int_{-\infty}^{\infty} u e^{-\lambda u^2} du}_{\substack{\text{odd function} \\ \text{symmetric limits} \\ \text{integral is zero}}} + \underbrace{Aa \int_{-\infty}^{\infty} e^{-\lambda u^2} du}_{Aa \frac{\pi^{1/2}}{\lambda^{1/2}}} = Aa \frac{\pi^{1/2}}{\lambda^{1/2}} = \frac{\lambda^{1/2}}{\pi^{1/2}} a \frac{\pi^{1/2}}{\lambda^{1/2}} = a$$

$$\langle x \rangle = a.$$

G1.3 pg 2

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 \rho(x)\, dx$$

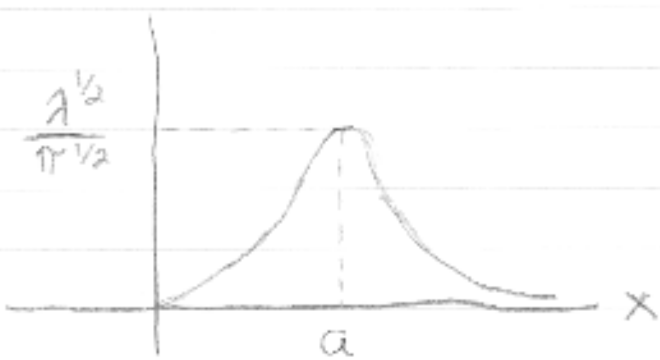
$$= A \int_{-\infty}^{\infty} x^2 e^{-\lambda(x-a)^2} dx$$

$$= A \int_{-\infty}^{\infty} (u+a)^2 e^{-\lambda u^2} du$$

$$= A \int_{-\infty}^{\infty} u^2 e^{-\lambda u^2} du + \underbrace{A \int_{-\infty}^{\infty} 2au\, e^{-\lambda u^2} du}_{\substack{\text{odd function} \\ \text{symmetric limits} \\ \text{integral} = 0}} + A \int_{-\infty}^{\infty} a^2 e^{-\lambda u^2} du$$

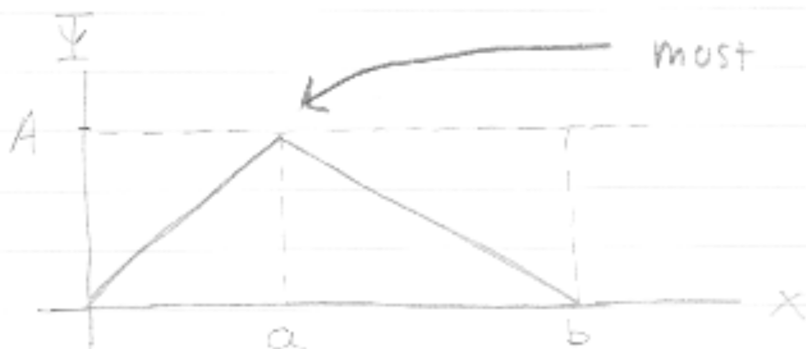
$$\langle x^2 \rangle = A \frac{\pi^{1/2}}{2\lambda^{3/2}} + A a^2 \frac{\pi^{1/2}}{\lambda^{1/2}} = \frac{1}{2\lambda} + a^2 = \frac{1 + 2\lambda a^2}{2\lambda}$$

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 = \frac{1}{2\lambda} + a^2 - a^2 = \frac{1}{2\lambda}$$



Griffith's 1.4

$$\Psi(x,0) = \begin{cases} A\dfrac{x}{a} & 0 \le x \le a \\ A\dfrac{(b-x)}{(b-a)} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

b) 

most probable $x = a$

a) $\displaystyle\int_{-\infty}^{\infty} \Psi^* \Psi \, dx =$

$$= A^2 \int_{-\infty}^{0} (0)^2 dx + \frac{A^2}{a^2}\int_0^a x^2 dx + \frac{A^2}{(b-a)^2}\int_a^b (b-x)^2 dx + A^2\int_b^{\infty} 0^2 dx$$

note $\displaystyle\int_a^b (b-x)^2 dx \qquad v = b-x \qquad dv = -dx$

$$\int (b-x)^2 dx \Rightarrow -\int v^2 dv = -\frac{v^3}{3}$$

$$\hookrightarrow \int (b-x)^2 dx = -\left.\frac{(b-x)^3}{3}\right|_a^b$$

$$= -0 + \frac{(b-a)^3}{3}$$

Griffiths 1.4 (a)

$$\therefore \int_{-\infty}^{\infty} \Psi^* \Psi \, dx = \frac{A^2}{a^2}\frac{a^3}{3} + \frac{A^2}{(b-a)^2}\frac{(b-a)^3}{3} = 1$$

to normalize

$$A^2\left[\frac{a}{3} + \frac{(b-a)}{3}\right] = 1$$

$$A^2 = 3/b \qquad A = \sqrt{\frac{3}{b}}$$

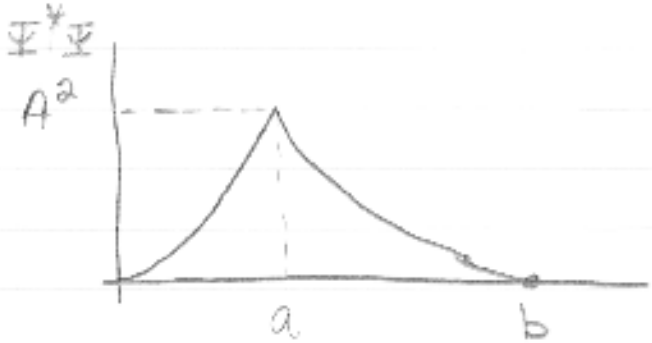
d) Probability that the particle is to the left of a is the area under the $\Psi^*\Psi$ graph from 0 to a

$$P_{0a} = \int_0^a \Psi^*\Psi \, dx \qquad (\text{use normalized } \Psi)$$

$$P_{0a} = \int_0^a A\frac{x}{a} \, A\frac{x}{a} \, dx$$

$$P_{0a} = \frac{A^2}{a^2}\int_0^a x^2 dx = \frac{A^2}{a^2}\frac{a^3}{3} = \frac{3}{b}\frac{a}{3} = \frac{a}{b}$$

G 1.4(3)



Note for this wave function $P_{0a} + P_{ab} = 1$

$$\therefore \quad P_{ab} = 1 - P_{0a} = 1 - P_{0a} = 1 - \frac{a}{b} = \frac{b-a}{b}$$

when $b = a$ there is no "second" region and the particle only exists in the region from 0 to a $\therefore$ $P_{0a} = 1$ note $P_{0a} = \frac{a}{b} = \frac{b}{b} = 1$

When $b = 2a$ the region from 0 to a and a to b are symmetric and we expect the probability that the particle is from 0 to a to be equal to the probability that it is from a to b $P_{0a} = P_{ab}$ $\therefore$ $P_{0a} = 1/2$

G1.4 (4)

$$P_{0a} = \frac{a}{b} = \frac{a}{2a} = \frac{1}{2}$$

e)

$$\langle x \rangle = \int_{-\infty}^{\infty} \Psi^* x \Psi \, dx$$

$$= \int_0^a x A^2 \frac{x^2}{a^2} \, dx + \int_a^b x \, A^2 \frac{(b-x)^2}{(b-a)^2} \, dx$$

Aside $\int x (b-x)^2 \, dx$

$$= \int (b^2 x - 2bx^2 + x^3) \, dx$$

$$= b^2 \frac{x^2}{2} - \frac{2}{3} b x^3 + \frac{x^4}{4}$$

Thus $\langle x \rangle = \frac{A^2}{a^2} \frac{a^4}{4} + \frac{A^2}{(b-a)^2} \left[\frac{b^2 b^2}{2} - \frac{2}{3} b b^3 + \frac{b^4}{4} - \frac{b^2 a^2}{2} + \frac{2}{3} b a^3 - \frac{a^4}{4} \right]$

$$\langle x \rangle = A^2 \left[\frac{a^2}{4} + \frac{1}{(b-a)^2} \left(\frac{b^4}{12} - \frac{b^2 a^2}{2} + \frac{2ba^3}{3} - \frac{a^4}{4} \right) \right]$$

G 1.4 (5)

$$\langle x \rangle = \frac{A^2}{(b-a)^2}\left[\frac{a^2(b-a)^2}{4} + \left(\frac{b^4}{12} - \frac{b^2a^2}{2} + \frac{2ba^3}{3} - \frac{a^4}{4}\right)\right]$$

$$\langle x \rangle = \frac{A^2}{(b-a)^2}\left[\frac{a^2b^2}{4} - \frac{2a^3b}{4} + \cancel{\frac{a^4}{4}} + \frac{b^4}{12} - \frac{b^2a^2}{2} + \frac{2ba^3}{3} - \cancel{\frac{a^4}{4}}\right]$$

$$\langle x \rangle = \frac{A^2}{(b-a)^2}\left[-\frac{b^2a^2}{4} + \frac{ba^3}{6} + \frac{b^4}{12}\right]$$

$$\langle x \rangle = \frac{A^2 b}{12(b-a)^2}\left[b^3 - 3ba^2 + 2a^3\right]$$

$$\langle x \rangle = \frac{A^2 b}{12(b-a)^2}\left[(b^2-a^2)(b-2a) - 2a^2b + 2ab^2\right]$$

$$\langle x \rangle = \frac{A^2 b}{12(b-a)^2}\left[(b-a)(b+a)(b-2a) + 2ab(b-a)\right]$$

$$\langle x \rangle = \frac{A^2 b}{12(b-a)}\left[b^2 - ab - 2a^2 + 2ab\right]$$

$$\langle x \rangle = \frac{A^2 b}{12(b-a)}\left[b^2 + ab - 2a^2\right]$$

G 1.4 (b)

$$\langle x\rangle = \frac{A^2 b}{12(b-a)} (b-a)(b+2a)$$

$$\langle x\rangle = \frac{A^2 b}{12}(b+2a) = \frac{3/b\ b}{12}(b+2a)$$

$$\langle x\rangle = \frac{1}{4}(b+2a)$$

$$\langle x^2\rangle = \int_{-\infty}^{\infty} \Psi^* x^2 \Psi\, dx$$

$$\langle x^2\rangle = \int_0^a x^2 A^2 \frac{x^2}{a^2}\, dx + \int_a^b x^2\ A^2 \frac{(b-x)^2}{(b-a)^2}\, dx$$

Aside $\int x^2 (b-x)^2\, dx$

$$= \int (b^2x^2 - 2bx^3 + x^4)\, dx$$

$$= \frac{b^2x^3}{3} - \frac{2bx^4}{4} + \frac{x^5}{5}$$

$$\langle x^2\rangle = \frac{A^2a^3}{5} + \frac{A^2}{(b-a)^2}\left[\frac{b^5}{3} - \frac{2b^5}{4} + \frac{b^5}{5} - \frac{b^2a^3}{3} + \frac{2ba^4}{4} - \frac{a^5}{5}\right]$$

G1.4 (7) note $\frac{1}{3} - \frac{2}{4} + \frac{1}{5} = \frac{20}{60} - \frac{30}{60} + \frac{12}{60} = \frac{2}{60} = \frac{1}{30}$

$$\langle x^2 \rangle = \frac{A^2 a^3}{5} + \frac{A^2}{(b-a)^2}\left[\frac{b^5}{30} - \frac{b^2a^3}{3} + \frac{2ba^4}{4} - \frac{a^5}{5}\right]$$

$$\langle x^2 \rangle = \frac{A^2}{(b-a)^2}\left[\frac{a^3b^2}{5} - \frac{2a^4b}{5} + \cancel{\frac{a^5}{5}} + \frac{b^5}{30} - \frac{b^2a^3}{3} + \frac{2ba^4}{4} - \cancel{\frac{a^5}{5}}\right]$$

$$\langle x^2 \rangle = \frac{A^2 b}{(b-a)^2}\left[\frac{b^4}{30} - \frac{2a^3b}{15} + \frac{1}{10}a^4\right]$$

$$\langle x^2 \rangle = \frac{A^2 b}{30(b-a)^2}\left[b^4 - 4a^3b + 3a^4\right]$$

$$\begin{array}{r|l}
 & b^3 + ab^2 + a^2b - 3a^3 \\
(b-a) & b^4 - 4a^3b + 3a^4 \\
 & -\ b^4 - ab^3 \\
 & +ab^3 - 4a^3b + 3a^4 \\
 & -[ab^3 - a^2b^2] \\
 & a^2b^2 - 4a^3b + 3a^4 \\
 & -[a^2b^2 - a^3b] \\
 & -3a^3b + 3a^4 \\
 & -\ -3a^3b + 3a^4 \\
 & 0
\end{array}$$

$$\therefore \langle x^2 \rangle = \frac{A^2 b}{30(b-a)^2}(b^3 + ab^2 + a^2b - 3a^3)(b-a)$$

G 1.4(8)

$$
\begin{array}{r}
b^2 + 2ab + 3a^2 \\
(b-a)\,\overline{\big)\, b^3 + ab^2 + a^2 b - 3a^3} \\
-(b^3 - ab^2) \\
\hline
2ab^2 + a^2 b - 3a^3 \\
-(2ab^2 - 2a^2 b) \\
\hline
+3a^2 b - 3a^3 \\
3a^2 b - 3a^3
\end{array}
$$

$$\therefore \langle x^2 \rangle = \frac{A^2 b}{30(b-a)^2}(b-a)(b-a)(b^2+2ab+3a^2)$$

$$\langle x^2 \rangle = \frac{A^2 b}{30}(b^2+2ab+3a^2)$$

$$\langle x^2 \rangle = \frac{(3/b)\, b}{30}(b^2+2ab+3a^2)$$

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$\sigma^2 = \frac{1}{10}(b^2+2ab+3a^2) - \left[\frac{1}{4}(b+2a)\right]^2$$

$$\sigma^2 = \frac{1}{10}(b^2+2ab+3a^2) - \frac{1}{16}(b^2+4ab+4a^2)$$

$$\sigma^2 = \frac{1}{80}(3b^2 - 4ab + 4a^2)$$

G1,4 (9)

$2a \doteq b$

$$\sigma^2 = \frac{1}{80}\left(12a^2 - 8a^2 + 4a^2\right)$$

$$\sigma^2 = \frac{1}{10}a^2 \qquad \sigma = \sqrt{\tfrac{1}{10}}\, a$$

$$\langle p \rangle = \int_{-\infty}^{\infty} \Psi^* \left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right) \Psi \, dx$$

$$\langle p \rangle = \int_0^a A\frac{x}{a}\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right) A\frac{x}{a}\, dx + \int_a^b \frac{A}{(b-a)}(b-x)\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\frac{A}{(b-a)}(b-x)\, dx$$

$$\langle p \rangle = \frac{A^2}{a^2}\frac{\hbar}{i}\int_0^a x\, dx \underset{\uparrow}{-} \frac{A^2}{(b-a)^2}\frac{\hbar}{i}\int_a^b (b-x)\, dx$$

derivative.

$$\langle p \rangle = \frac{A^2}{a^2}\frac{\hbar}{i}\frac{a^2}{2} - \frac{A^2}{(b-a)^2}\frac{\hbar}{i}\left(b^2 - ba - \frac{b^2}{2} + \frac{a^2}{2}\right)$$

$$\langle p \rangle = \frac{A^2}{2}\frac{\hbar}{i} - \frac{A^2}{2(b-a)^2}\frac{\hbar}{i}\left(b^2 - ba + a^2\right) \leftarrow (b-a)^2$$

$$\langle p \rangle = A^2\frac{\hbar}{i}\left(\frac{1}{2} + \frac{1}{2}\right) = \frac{3}{b}\frac{\hbar}{i}$$

61.4 (10)

$$\langle p^2 \rangle = \int_{-\infty}^{\infty} \Psi^* \left(\frac{\hbar^2}{i^2} \frac{\partial^2}{\partial x^2} \right) \Psi \, dx$$

$$= -\frac{\hbar^2 A^2}{a^2} \int_0^a x \frac{\partial^2}{\partial x^2} x \, dx - \frac{\hbar^2 A^2}{(b-a)^2} \int_a^b (b-x) \frac{\partial^2}{\partial x^2} (b-x) \, dx$$

$$\langle p^2 \rangle = 0$$

$$\sigma_p^2 = \langle p^2 \rangle - \langle p \rangle^2 = 0 - \left(\frac{3}{b} \frac{\hbar}{i} \right)^2 = \left(\frac{9\hbar^2}{b^2} \right)$$

For $\sigma_x \sigma_p \geq \hbar/2$ for $2a = b$ ...

$$\left(\sqrt{\frac{1}{10}} \, a \right) \left(\frac{3\hbar}{b} \right) = \sqrt{\frac{9}{10}} \left(\frac{a}{b} \right) \hbar \geq \hbar/2$$