1b)

$$\bar{E} = \frac{\int_0^\infty E \frac{1}{kT} e^{-E/kT} dE}{\int_0^\infty \frac{1}{kT} e^{-E/kT} dE}$$

$$\frac{1}{kT}\int_0^\infty e^{-E/kT} dE = \frac{1}{kT}(-kT)\, e^{-E/kT}\Big|_0^\infty$$

$$= -\left(e^{-\infty/kT} - e^0\right) = 1$$

$$\frac{1}{kT}\int_0^\infty \underbrace{E}_{f}\, \underbrace{e^{-E/kT}\, dE}_{dg}$$

$$d(gf) = dg\, f + g\, df$$

$$\therefore \int dg\, f = gf - \int g\, df$$

$$dg = e^{-E/kT} dE \qquad f = E$$
$$g = (-kT)\, e^{-E/kT} \qquad df = dE$$

$$\int_0^\infty E e^{-E/kT} = \underbrace{\left(-kTe^{-E/kT} E\right)\Big|_0^\infty}_{\text{This equals zero at both limits}} - \int -kTe^{-E/kT} dE$$

$$= kT\int_0^\infty e^{-E/kT} dE$$

$$= kT(-kT)\, e^{-E/kT}\Big|_0^\infty$$

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$$= -(KT)^2\left(e^{-\infty/KT} - e^{0/KT}\right) = (KT)^2$$

$$\therefore \bar{E} = \int_0^\infty E \frac{1}{kT} e^{-E/kT}\, dE$$

$$= \frac{1}{kT}\int_0^\infty E e^{-E/kT}\, dE = \frac{1}{kT}(KT)^2 = kT$$

17)

$$\rho_T(\nu) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/kT}-1}$$

$$R_T = \int_0^\infty R_T(\nu)\, d\nu$$

$$R_T = \int_0^\infty \frac{c}{4} \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/kT}-1}\, d\nu$$

$$R_T = \frac{2\pi h}{c^2} \int_0^\infty \frac{\nu^3}{e^{h\nu/kT}-1}\, d\nu$$

$$\underline{q \equiv h\nu/kT} \qquad dq = \frac{h}{kT}\, d\nu \qquad \underline{d\nu = \frac{kT}{h}\, dq}$$

$$q^3 = \frac{h^3\nu^3}{(kT)^3} \qquad \nu^3 = \frac{q^3(kT)^3}{h^3}$$

$$\therefore \quad R_T = \frac{2\pi h}{c^2} \frac{(kT)^3}{h^3} \frac{kT}{h} \underbrace{\int_0^\infty \frac{q^3\, dq}{e^q-1}}_{= \frac{\pi^4}{15}}$$

$$R_T = \frac{2\pi k^4 \pi^4}{c^2 h^3\, 15} T^4 = \sigma T^4$$

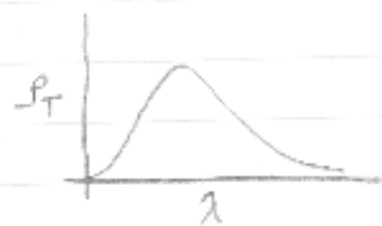
18)

$$\rho_T(\nu)\,d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT}-1}\,d\nu$$

$$\nu = c/\lambda \qquad d\nu = -\frac{c}{\lambda^2}\,d\lambda$$

$$\rho_T(\nu)\,d\nu = \frac{8\pi h}{c^3}\frac{c^3/\lambda^3}{e^{hc/kT\lambda}-1}\left(\frac{-c}{\lambda^2}\right)d\lambda = \rho_T(\lambda)\,d\lambda$$

$$\rho_T(\lambda)\,d\lambda = 8\pi hc\,\frac{1}{\lambda^5}\,\frac{d\lambda}{e^{hc/kT\lambda}-1}$$



$$\frac{d\left(\rho_T(\lambda)\right)}{d\lambda} = 8\pi hc\,\frac{d}{d\lambda}\left(\frac{1}{\lambda^5}\frac{1}{e^{hc/kT\lambda}-1}\right) = 0$$

$$= -\frac{5}{\lambda^6}\frac{1}{e^{hc/kT\lambda}-1} + \frac{1}{\lambda^5}\frac{\frac{hc}{kT}\frac{1}{\lambda^2}e^{hc/kT\lambda}}{\left(e^{hc/kT\lambda}-1\right)^2} = 0$$

$$5\left(e^{hc/kT\lambda}-1\right) = \frac{hc}{kT\lambda}\,e^{hc/kT\lambda}$$

$$x \equiv \frac{hc}{kT\lambda}$$

$$\therefore\quad 5\left(e^{x}-1\right) = x e^{x}$$

$$5\left(1-e^{-x}\right) = x$$

$$\frac{x}{5} + e^{-x} = 1 \qquad \therefore\ x = 4.965$$

18) page 2

$$\frac{hc}{KT\lambda_{max}} = 4.965$$

$$\lambda_{max} T = \frac{hc}{K\, 4.965} = \frac{1}{4.965} \frac{hc}{K}$$