An Analysis of Initial Velocity Error in Atwood's Machine

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Abstract

Atwood's Machine has traditionally been used in introductory physics classes to introduce and reinforce the concepts of gravity, uniform acceleration, and Newton's laws of motion. While several techniques for using the apparatus have been developed for use with different levels of students, there has not been a systematic analysis of initial velocity error. This study presents methods for measuring and tracking this main source of error through the standard Atwood protocol.

Introduction

Atwood's Machine is an experimental set-up used to teach elementary concepts in mechanics in an introductory physics laboratory. It consists of two unequal masses hung over a pulley by a light string. (Shown at right.)

A typical experiment involves measuring the time (*t*) the heavier mass requires to fall a known distance, *y*. The standard kinematic formula, assuming constant acceleration is:

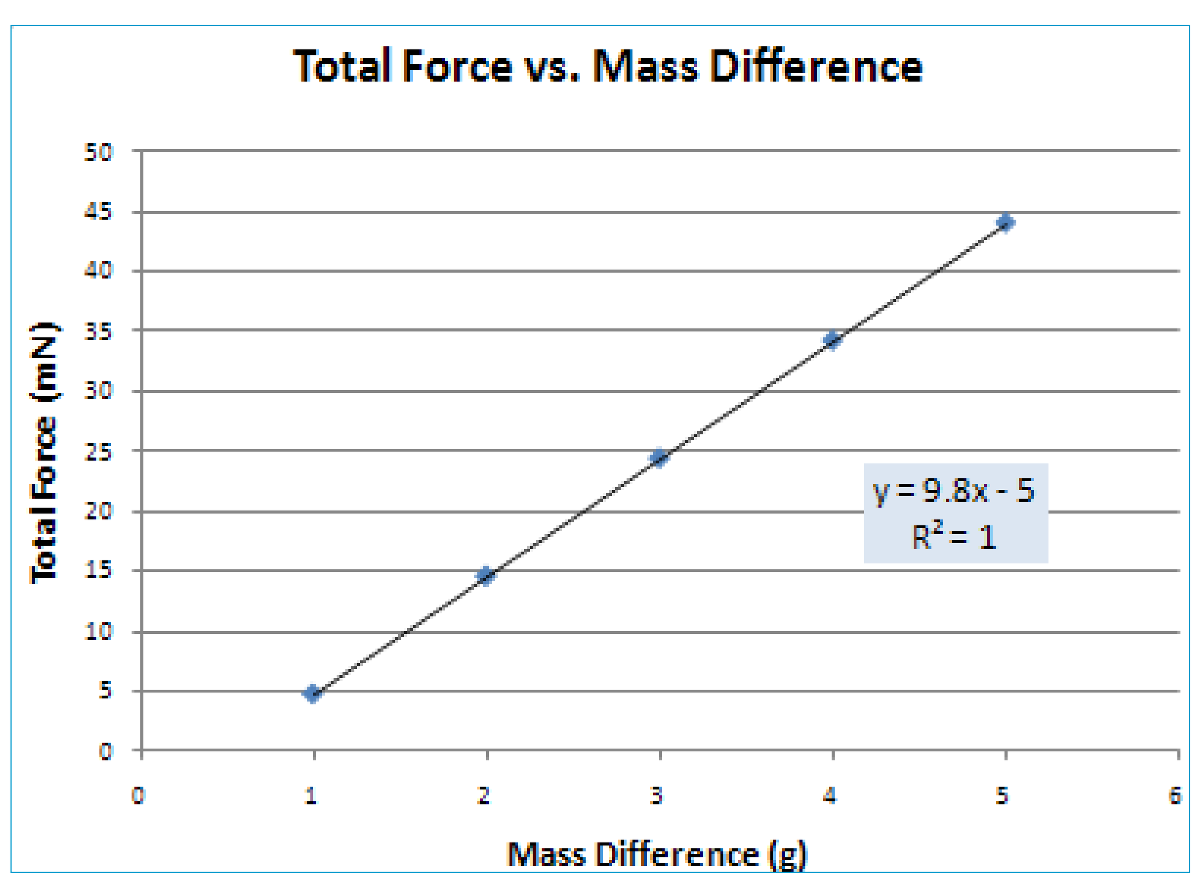
$$y = v_i t + \frac{1}{2}at^2$$

The acceleration is calculated as follows, assuming the objects start from rest:

$$a = \frac{2y}{t^2}$$

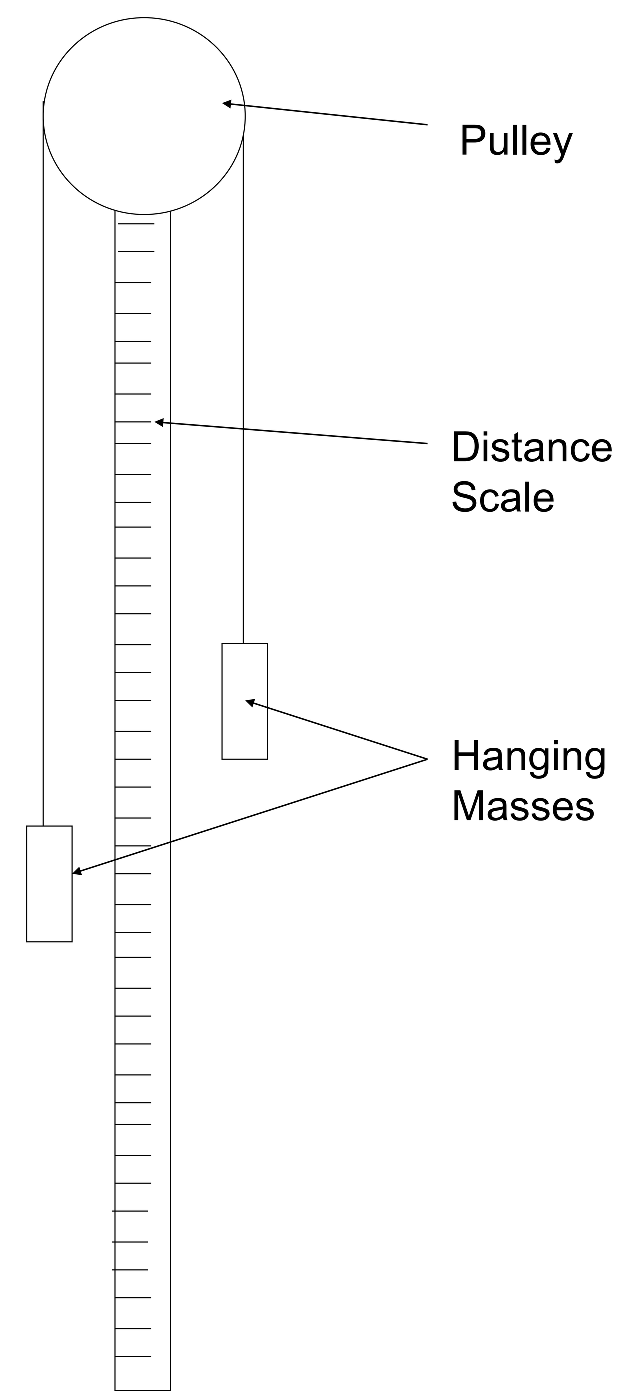
If the the mass difference is varied while the total mass is kept constant, these time measurements and acceleration calculations conspire with Newton's Second Law of motion to produce a linear graph with a slope that equals the acceleration due to gravity and an intercept that corresponds to the effective frictional "force" (most of which is assumed to be frictional torque within the pulley).

$$m_T a = (m_2 - m_1)g - F_f$$



Note: As indicated in Shea [4], there are basic three levels of analysis:
1) Assuming friction is negligible,
2) Including friction, but neglecting the rotational inertia of the pulley, and
3) Including friction as well as rotational inertia.

This study uses the second level of complexity, but may be applied to any of the three methods.



Error in Atwood's Machine

Atwood's machine errors include:
1) Timing errors
2) Variable friction
3) Errors in mass and distance measurement
4) Initial velocity errors

Timing errors maybe be reduced by using smart pulleys or repeated stopwatch measurements, although systematic errors may still occur.

Variable friction may be overcome by using pulleys with a point bearing [4].

The errors in mass and distance measurement are assumed to be small.

This study focuses on initial velocity errors. As indicated in the Introduction, the hanging masses are released from rest. However, the person releasing these masses invariably imparts slight, but non-negligible, non-zero initial velocities, estimated to be in the range of one to two centimeters per second. While this tends to be less than one percent of the average velocity of the masses during a trial, it can result in a much more substantial error in the extracted values of gravitational acceleration and friction.

Initial Velocity Error

Initial velocity affects the motion of the hanging masses as follows:

$$y = v_i t + \frac{1}{2}at^2$$

The measured time is now:

$$t = \frac{-v_i + \sqrt{v_i^2 + 2ay}}{a}$$

Using the binomial approximation on the radical (assuming the initial velocity is small compared to the average velocity), we see that the time discrepancy is proportional to the initial velocity, and is more pronounced for small accelerations.

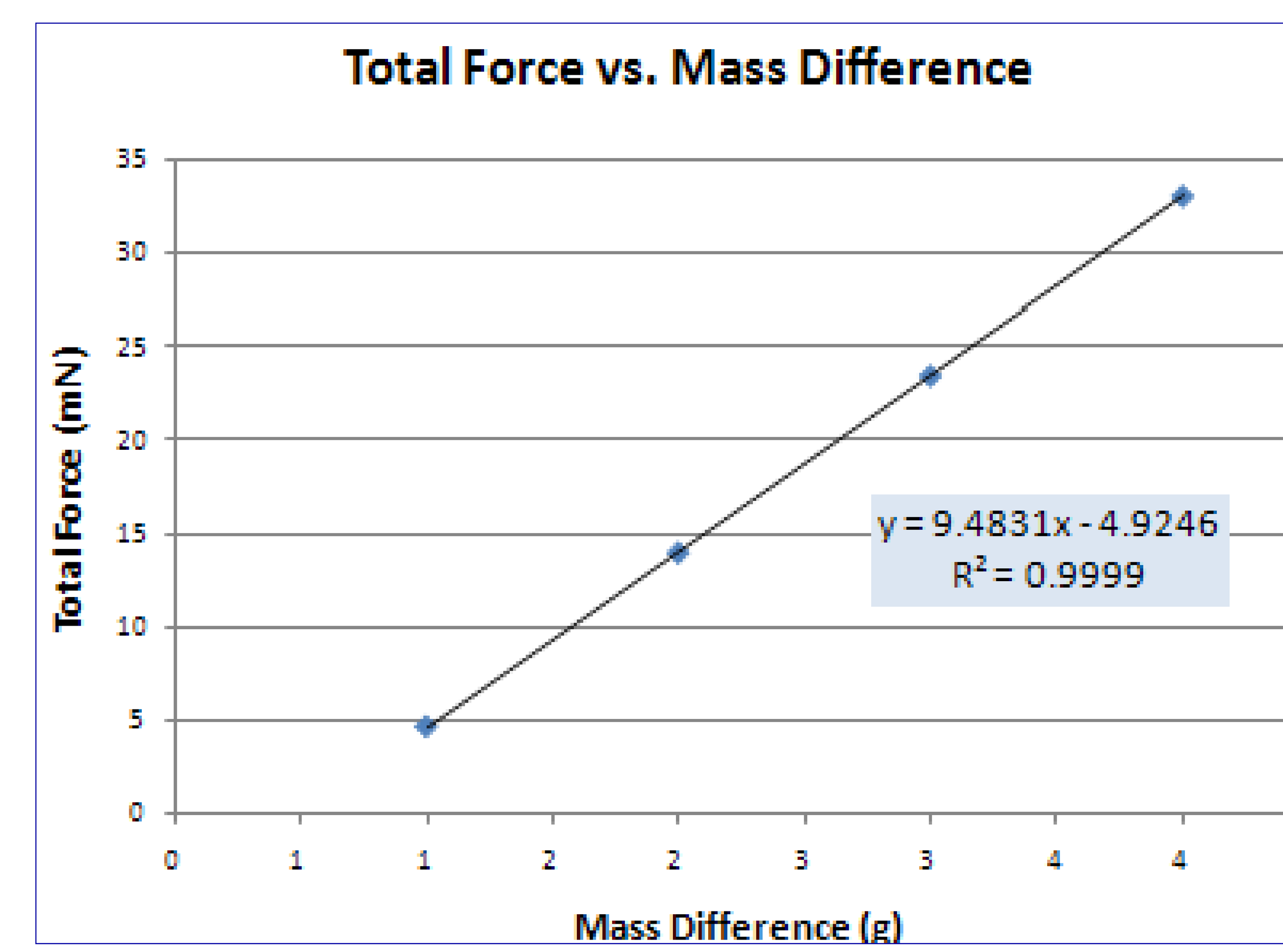
$$\delta t \equiv t - t_0 \approx -\frac{v_i}{a}$$

Simulated Results

Simulations using *MSExcel* were used to determine the effect of random initial velocity error in the range of 2.0 cm/s.

A typical result is shown at right, where the gravitational acceleration is found to be over 3% off. The degree of fit of 0.9999 provides a disturbingly false sense of precision.

Additional simulations produced the following preliminary conclusions:
1) The errors in gravitational acceleration were off by up to five percent.
2) The errors tend to be reduced as the number of data points is increased, although *large errors may still occur when using large numbers of data points*.
3) Errors in frictional torque are similar to those in *g*.



Future Work

• Estimates of initial velocity error are not yet definitive in a typical Atwood scenario. Measurements using a high-speed video system will produce better results.
•While this study includes random initial velocity error, systematic errors (probably skewed in the upward direction) must be investigated.
•Systematic (stopwatch) timing errors are also suspected. The effect of these errors must be compared to that of initial velocity error.

References

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[4] Shea, R. and French, P., "Atwood's Machine: Methods to Success," Student Research Day, SUNY College at Oneonta, April 2009

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