**Introduction**

Atwood’s Machine is an experimental set-up used to teach elementary concepts in mechanics in an introductory physics laboratory. It consists of two unequal masses hung over a pulley by a light string. (Shown at right.) A typical experiment involves measuring the time \( t \) the heavier mass requires to fall a known distance, \( y \). The standard kinematic formula, assuming constant acceleration is:

\[
y = y_f + \frac{1}{2}at^2
\]

The acceleration is calculated as follows, assuming the objects start from rest:

\[
a = \frac{2y}{t^2}
\]

If the mass difference is varied while the total mass is kept constant, these time measurements and acceleration calculations conspire with Newton’s Second Law of motion to produce a linear graph with a slope that equals the acceleration due to gravity and an intercept that corresponds to the effective frictional “force” (most of which is assumed to be frictional torque within the pulley).

\[
m_1a = (m_2 - m_1)g - F_f
\]

**The String Effect**

To account for the string effect, where \( \lambda \) = linear mass density, we use Newton’s Second Law of motion to describe the motion of each side, adding mass corresponding to the amount of string. In order to focus on the string effect, we assume an ideal (massless and frictionless) “point” pulley.

\[
(m_1 + \lambda x)g - T = (m_1 + \lambda x)a
\]

\[
T - (m_1 + \lambda(L-x))g = (m_1 + \lambda(L-x))a
\]

which simplify to

\[
x - \alpha^2x = b
\]

where \( \alpha^2 = -\frac{2\lambda g}{m_1 + \lambda L} \)

and

\[
b = \frac{(m_1 - m_2 - \lambda L)g}{m_1 + m_2 + \lambda L}
\]

With the initial conditions of \( x = 0 \), the solution is:

\[
x = \frac{b}{\alpha^2} \left( \cosh(\alpha t) - 1 \right)
\]

In preliminary trials, with small mass differences and heavy-duty string, this effect can result in a tripling of the acceleration, thus overwhelming the expected dependence on speed or load of the pulley friction.

**Methodology**

The following methods will be employed to overcome the string effect.

1. The load refers to the total effective mass of the system, or

\[
m_t = m_1 + m_2 + \lambda L \cdot \frac{t}{r^2}
\]

2. The smart pulley system uses a photogate to detect the rotation of the pulley wheel, which is carefully calibrated to circumferential position. The position versus time data will be “binned,” and speed and acceleration will be extracted using linear and quadratic fits.

3. No-load measurements – the string effect is removed, and a simple acceleration versus speed graph is obtained from the position versus time data using the smart pulley system.

4. Under non-zero load (normal operation of Atwood’s Machine), random error or noise makes “bining” the data problematic. However, with enough trials, we hope that the string effect may be subtracted out, yielding acceleration versus speed data for a variety of loads. At each load value, the mass-difference will be adjusted in order to expand the velocity range.

**Abstract**

Atwood’s Machine is a simple pulley system developed in the 18th century to verify the laws of motion. It is still a staple in the introductory teaching laboratory, as it can illuminate many physical effects, as well as mathematical techniques. This study, which builds on faculty and student research projects over the past few years at SUNY Oneonta, will help to expose the intricacies of measuring the friction in Atwood’s Machine, and whether and how this torque varies with the speed of the pulley.

**References**


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